THE UNIVERSITY OF SZCZECIN
FACULTY OF ECONOMICS AND MANAGEMENT
Institute of Econometrics and Statistics

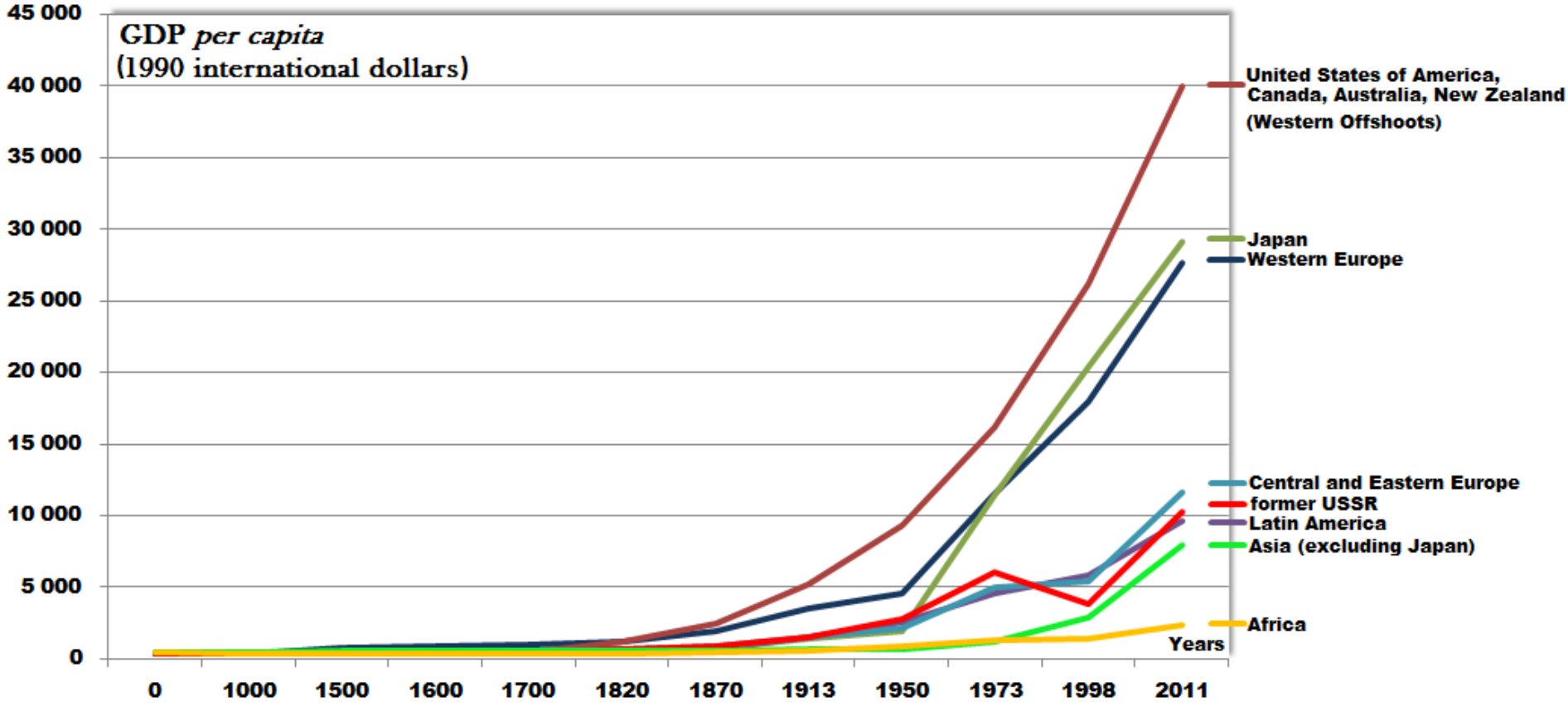
Modelling the Growth of Nations. Is Gender Equality an Important Factor of the Long-Run Economic Growth?

dr Christian Lis
Institute of Econometrics and Statistics
Department of Statistics

The list of contents

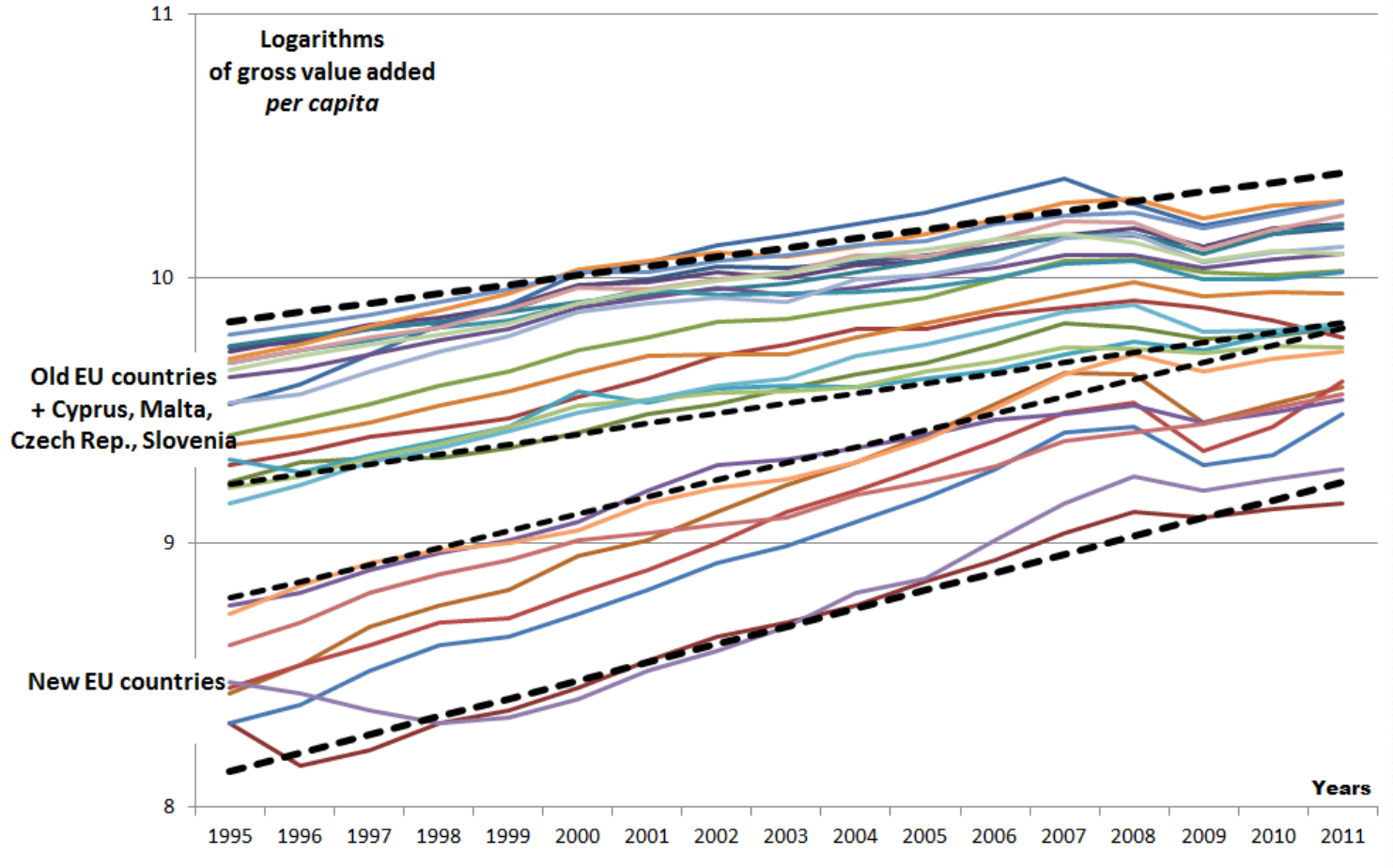
1. Divergence or convergence.
2. Long-run economic growth theories.
3. Growth sources.
4. How to measure gender equality?

The Great Divergence



Source: Years 1–1998: A. Maddison, *The World Economy. A Millennial Perspective*, Development Centre of the Organisation for Economic Co-Operation and Development, 2001, p. 264; Years 1998–2011: Own reworking based on IMF data published in *World Economic Outlook Database*, Apr. 2012.

European Union as a convergence club



Source: Ch. Lis, *Gross Value Added and Its Significance in the Capital Formation in Relation to the Growth and Convergence Theories. A Taxonomical Approach*, Volumina.pl, Szczecin 2013, p. 237;

The Stylized Facts of Growth

1. Output per worker grows continuously, with no tendency for the rate of growth of productivity to decline (Kaldor).
2. The capital–labour ratio shows continuous growth (Kaldor).
3. The rate of return on capital is stable (Kaldor).
4. The capital–output ratio is stable (Kaldor).
5. The shares of labour and capital in GDP remain stable (Kaldor).
6. We observe significant variation in the rate of growth of productivity across countries (Kaldor).

Source: N. Kaldor, *Capital Accumulation and Economic Growth*, in F.A. Lutz and D.C. Hague (eds), *The Theory of Capital*, New York: St Martin's Press, 1961;

The Stylized Facts of Growth (*cont.*)

7. In a broad cross-section of countries the average growth rate is uncorrelated with the level of per capita income (Romer).
8. Growth is positively correlated with the volume of international trade (Romer).
9. Growth rates are negatively correlated with population growth (Romer).
10. Growth accounting research always finds a 'residual'; that is, accumulation of factor inputs alone cannot account for growth (Romer).
11. High-income countries attract both skilled and unskilled workers (Romer).

Source: P.M. Romer, *Capital Accumulation in the Theory of Long-Run Growth*, in R.J. Barro (ed.), *Modern Business Cycle Theory*, Cambridge, MA: Harvard University Press, 1989;

The Stylized Facts of Growth (*cont.*)

12. There is enormous variation in income per capita across countries (Jones).
13. Growth rates for the world as a whole, and for individual countries, vary substantially over time (Jones).
14. The relative position of any country in the world distribution of income can change (Jones).
15. **There is positive correlation between GDP *per capita* and gender equality both across countries and over time.**

Source: C.I. Jones, *Introduction to Economic Growth*, 2nd edition, New York: W.W. Norton., 2001;

Long-run economic growth theories

1. Neoclassical growth theory.

Adelman model, Harrod-Domar model, Solow (Solow-Swan) model, Cobb-Douglas model, Solow-Minhas-Arrow-Chenery (SMAC) model, Brown-de Cani model (CES), Inada model, Uzawa model, R. Sato model, K. Sato model, Takayama model, Ramsey-Cass-Koopmans model (RCK), Mankiw-Romer-Weil model (MRW) and many others;

2. Endogenous growth theory.

Uzawa model (宇沢弘文 Hirofumi Uzawa), Shell model, K. Sato model, P.M. Romer model, R.E. Lucas model, Uzawa-Lucas model, Heckman model, Rosen model, Grossman-Helpman model, Aghion-Howitt model, Jones model, Jones-Kortum-Segerstrom model, Eicher-Turnovsky model, Barro model and many others;

3. Real business cycle theory.

F.E. Kydland, E.C. Prescott, C.R. Nelson, C.I. Plosser, O.J. Blanchard, S. Fischer, and others

4. Sustainable development theory.

Harrod-Domar model

Assumptions

- product is formed by two sectors: companies and households;
- exogenous labour growth rate is constant (n);
- ratios K_t/L_t and K_t/Y_t are constant;
- product is the sum of consumption and savings: $Y_t = C_t + S_t$;
- in two-sector economy all savings are invested, thus $Y_t = C_t + I_t$;
- future capital is the sum of investment and capital from the previous period reduced by its depreciation:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

If $\Delta K/\Delta Y = \nu$ and $S_t = I_t = sY_t$, then

$$\nu Y_{t+1} = (1-\delta)\ \nu Y_t + sY_t,$$

After transformation: $Y_{t+1} - Y_t = (s/\nu - \delta)\ Y_t,$

$$(Y_{t+1} - Y_t)/\ Y_t = (s/\nu - \delta) = G$$

Solow model

The key assumptions of the Solow model are:

- for simplicity it is assumed that the economy consists of one sector producing one type of commodity that can be used for either investment or consumption purposes;
- the economy is closed to international transactions and the government sector is ignored;
- all output that is saved is invested;
- Solow abandons the Harrod–Domar assumptions of a fixed capital–output ratio (K/Y) and fixed capital–labour ratio (K/L);
- the rate of technological progress, population growth and the depreciation rate of the capital stock are all determined exogenously.

Model is built around the neoclassical aggregate production function and focuses on the *proximate* causes of growth:

$$Y_t = f(K_t, L_t, A_t, U_t).$$

The particular case of the Solow model

- Original Cobb-Douglas model

  (C.W. Cobb, P.H. Douglas, *A Theory of Production*, The American Economic Review, Vol. 18, No. 1, Supplement, *Papers and Proceedings of the Fortieth Annual Meeting of the American Economic Association*, American Economic Association (Mar. 1928), p. 139- 165)

$$P' = bL^k C^{1-k}$$ *(Original notation has been kept)*

- Modified Cobb-Douglas model (*inconstant elasticity of substitution*)

$$Y_t = \alpha_0 K_t^{\alpha_1} L_t^{\alpha_2} e^{U_t}, \quad (\alpha_0 > 0; \quad 0 < \alpha_1, \alpha_2 < 1)$$

- Modified Cobb-Douglas model with exogenous technological progress

$$Y_t = \alpha_0 K_t^{\alpha_1} L_t^{\alpha_2} e^{\gamma t} e^{U_t}$$

Uzawa model – an endogenous model of economic growth

The aggregate production function at each moment of time *t* can be written as follows:

$$Y_t = F\left(K_t, A_t \cdot L_{Pt}\right),$$

where the state of technological knowledge at time *t* is represented by the efficiency in labour A_t.

It is assumed that various activities in the form education, health, construction and maintenance of public goods, etc., which results in an improvement in labour efficiency A_t, are put together as one sector to be referred to as the educational sector.

Uzawa proposed the model $y = f(k)$ for output *per capita* $y = Y/L$, that is related to the capital-labour ratio $k = K/L$, namely

$$f\left(k_t\right) = \frac{F\left(K_t, L_t\right)}{L_t} = F\left(k_t, 1\right),$$

where the function $y = f(k)$ is continuous, twice-differentiable, positive, increasing and concave.

Uzawa model (*cont.*)

Labour allocation to the productive and educational sector has to be estimated.

It is assumed that the higher fraction of labour in the educational sector, the higher level of production in the economy.

Everyone can not be employed in the educational sector.

The problem is to find a time path of the economy over which the discounted sum of consumption *per capita*

$$\int_0^{+\infty} \frac{C_t}{L_t} e^{-\delta t} dt = \int_0^{+\infty} (1 - s_t) y_t e^{-\delta t} dt$$

is maximized among all feasible paths resulting from the given initial capital stock K_0 and labour efficiency A_0.

The problem can be solved thanks to Pontryagin's Maximum Principle*.

* L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Interscience Publishers, New York, London 1962;
H. Uzawa, *Optimum Technical Change in An Aggregative Model of Economic Growth*, International Economic Review, Vol. 6, No. 1. (Jan., 1965), s. 21

I. Adelman model

The production function can be expressed as equation:

$$Y_t = f\left(K_t, N_t, L_t, A_t, S_t\right),$$

where:

K_t - capital stock,

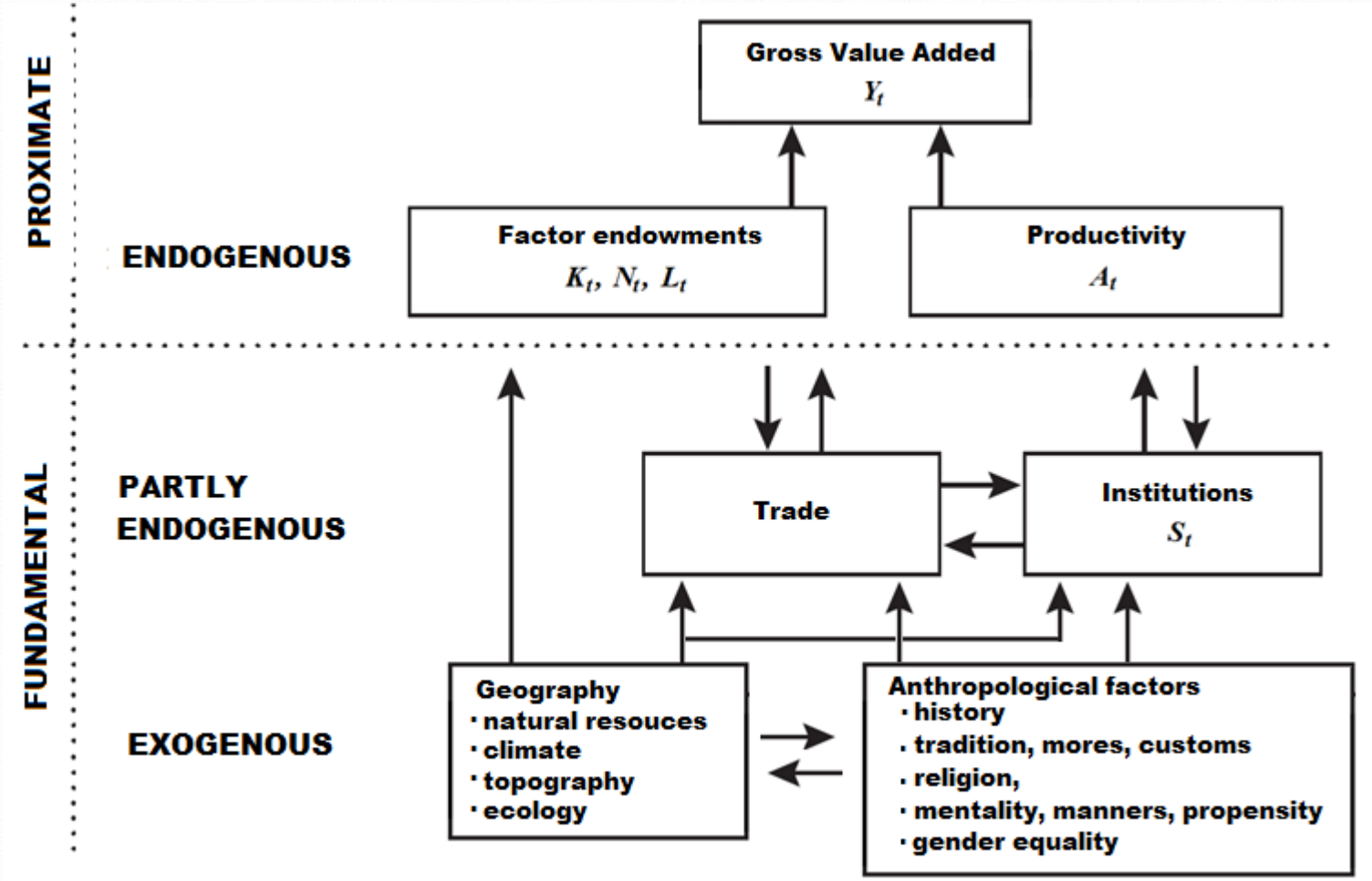
N_t - natural resources (geography),

L_t - represents labour resources,

A_t - denotes an economy's stock of applied knowledge,

S_t - represents what Adelman calls the 'sociocultural milieu', (and Abramovitz (1986) more recently has called 'social capability').

Growth sources



Source: Own rework based on D. Rodrik (ed.), *In Search of Prosperity, „Chapter 1 - Introduction. What Do We Learn From Country Narratives?"*, Princeton University Press, 2003, p. 6.

How to measure gender equality?

In a study conducted by the United Nations Development Programme (UNDP) two indicators are used in order to evaluate the participation of women in social development, i.e. Gender-related Development Index – GDI (since 2010 – Gender Inequality Index GII) and Gender Empowerment Measure – GEM.

ID	EU Countries	Gender-related development index (GDI)	Life expectancy at birth (years) 2007		Adult literacy rate (% aged 15 and above) 1999–2007		Combined gross enrolment ratio in education (%) 2007		Estimated earned income (PPP US$) 2007	
			Female	Male	Female	Male	Female	Male	Female	Male
1	Sweden	**0,956**	83,0	78,6	..	..	99,0	89,8	29 476	44 071
2	France	**0,956**	84,5	77,4	..	..	97,4	93,5	25 677	42 091
3	Netherlands	**0,954**	81,9	77,6	..	..	97,1	97,9	31 048	46 509
4	Finland	**0,954**	82,8	76,0	..	..	105,1	97,9	29 160	40 126
5	Spain	**0,949**	84,0	77,5	97,3	98,6	99,9	93,3	21 817	41 597
6	Ireland	**0,948**	82,0	77,3	..	..	99,1	96,2	31 978	57 320
7	Belgium	**0,948**	82,4	76,5	..	..	95,9	92,8	27 333	42 866
8	Denmark	**0,947**	80,5	75,9	..	..	105,3	97,6	30 745	41 630
9	Italy	**0,945**	84,0	78,1	98,6	99,1	94,7	89,1	20 152	41 158
10	Luxembourg	**0,943**	82,0	76,5	..	..	94,7	94,0	57 676	101 855
11	United Kingdom	**0,943**	81,5	77,1	..	..	92,8	85,9	28 421	42 133
12	Germany	**0,939**	82,3	77,0	..	..	87,5	88,6	25 691	43 515
13	Greece	**0,936**	81,3	76,9	96,0	98,2	103,2	100,1	19 218	38 002
14	Austria	**0,930**	82,5	77,0	..	..	92,1	89,0	21 380	54 037
15	Slovenia	**0,927**	81,7	74,4	99,6	99,7	98,1	87,7	20 427	33 398
16	Cyprus	**0,911**	81,9	77,3	96,6	99,0	77,8	77,3	18 307	31 625
17	Portugal	**0,907**	81,8	75,3	93,3	96,6	91,6	86,2	17 154	28 762
18	Czech Republic	**0,900**	79,4	73,2	..	..	85,1	81,9	17 706	30 909
19	Malta	**0,895**	81,3	77,7	93,5	91,2	81,7	81,0	14 458	31 812
20	Estonia	**0,882**	78,3	67,3	99,8	99,8	98,2	84,6	16 256	25 169
21	Hungary	**0,879**	77,3	69,2	98,8	99,0	94,0	86,6	16 143	21 625
22	Poland	**0,877**	79,7	71,3	99,0	99,6	91,4	84,2	11 957	20 292
23	Slovakia	**0,877**	78,5	70,7	..	..	83,1	77,9	14 790	25 684
24	Lithuania	**0,869**	77,7	65,9	99,7	99,7	97,6	87,2	14 633	20 944
25	Latvia	**0,865**	77,1	67,1	99,8	99,8	97,5	83,2	13 403	19 860
26	Bulgaria	**0,839**	76,7	69,6	97,9	98,6	82,9	81,8	9 132	13 439
27	Romania	**0,836**	76,1	69,0	96,9	98,3	81,7	76,7	10 053	14 808

Source: *Human Development Report 2009, Overcoming barriers: Human mobility and development*, Published for the United Nations Development Programme (UNDP), 2009, 1 UN Plaza, New York, NY 10017, USA

Rank	EU Countries	Gender empowerment measure (GEM)	Seats in parliament held by women	Female legislators, senior officials and managers	Female professional and technical workers	Ratio of estimated female to male earned income	Year women received right to		Year a woman became Presiding Officer of parliament or of one of its houses for the first time	Women in ministerial positions
							vote	stand for election		
		Value	(% of total)	(% of total)	(% of total)					(% of total)
1	Sweden	**0,909**	47	32	51	0,67	1919, 1921	1919, 1921	1991	48
2	Finland	**0,902**	42	29	55	0,73	1906	1906	1991	58
3	Denmark	**0,896**	38	28	52	0,74	1915	1915	1950	37
4	Netherlands	**0,882**	39	28	50	0,67	1919	1917	1998	33
5	Belgium	**0,874**	36	32	49	0,64	1919, 1948	1921	2004	23
6	Germany	**0,852**	31	38	50	0,59	1918	1918	1972	33
7	Spain	**0,835**	34	32	49	0,52	1931	1931	1999	44
8	United Kingdom	**0,790**	20	34	47	0,67	1918, 1928	1918, 1928	1992	23
9	France	**0,779**	20	38	48	0,61	1944	1944	..	47
10	Portugal	**0,753**	28	32	51	0,60	1931, 1976	1931, 1976	..	13
11	Austria	**0,744**	27	27	48	0,40	1918	1918	1927	38
12	Italy	**0,741**	20	34	47	0,49	1945	1945	1979	24
13	Luxembourg	**0,741**	23	..	..	0,57	1919	1919	1989	14
14	Ireland	**0,722**	15	31	53	0,56	1918, 1928	1918, 1928	1982	21
15	Greece	**0,677**	15	28	49	0,51	1952	1952	2004	12
16	Estonia	**0,665**	21	34	69	0,65	1918	1918	2003	23
17	Czech Republic	**0,664**	16	29	53	0,57	1920	1920	1998	13
18	Slovakia	**0,663**	19	31	58	0,58	1920	1920	..	13
19	Latvia	**0,648**	20	41	66	0,67	1918	1918	1995	22
20	Slovenia	**0,641**	10	34	56	0,61	1946	1946	..	18
21	Poland	**0,631**	18	36	60	0,59	1918	1918	1997	26
22	Lithuania	**0,628**	18	38	70	0,70	1919	1919	..	23
23	Bulgaria	**0,613**	22	31	61	0,68	1937, 1945	1945	..	24
24	Cyprus	**0,603**	14	15	48	0,58	1960	1960	..	18
25	Hungary	**0,590**	11	35	60	0,75	1918, 1945	1918, 1945	1963	21
26	Malta	**0,531**	9	19	41	0,45	1947	1947	1996	15
27	Romania	**0,512**	10	28	56	0,68	1929, 1946	1929, 1946	2008	0

Source: *Human Development Report 2009, Overcoming barriers: Human mobility and development*, Published for the United Nations Development Programme (UNDP), 2009, 1 UN Plaza, New York, NY 10017, USA

Variables selection

4. *Gender equality* variables:
   - GDI – *Gender-related Development Index* – X_{32i};
   - *GEM – Gender Empowerment Measure* – X_{33i};
   - Difference in life expectancy between men and women (in years) – X_{34i};
   - At-risk-of-poverty rate, males – X_{35i};
   - At-risk-of-poverty rate, females – X_{36i};
   - Relation between avarage wage for women and men (%) – X_{37i};
   - Difference in healthy life years for women and men – X_{38i};

Data set

Country	X_{32i}	X_{33i}	X_{34i}	X_{35i}	X_{36i}	X_{37i}	X_{38i}
Belgium	0,948	0,874	5,5	13,6	15,9	86,5	0,4
Bulgaria	0,839	0,613	7,3	19,8	22,9	87,1	3,6
Czech Republic	0,900	0,664	6,5	8,0	10,1	75,7	2,1
Denmark	0,947	0,896	4,5	11,7	12,0	78,7	-1,6
Germany	0,939	0,852	5,0	14,2	16,2	76,9	1,6
Estonia	0,882	0,665	10,8	16,5	22,0	68,7	4,5
Ireland	0,948	0,722	4,8	14,5	16,4	76,1	1,8
Greece	0,936	0,677	4,7	19,6	20,7	76,2	0,4
Spain	0,949	0,835	6,3	18,3	21,0	79,7	-0,5
France	0,956	0,779	7,2	12,7	14,0	81,8	1,8
Italy	0,945	0,741	5,5	17,1	20,1	82,1	-0,9
Cyprus	0,911	0,603	4,6	14,0	18,3	74,9	0,6
Latvia	0,865	0,648	10,8	23,1	27,7	83,8	2,6
Lithuania	0,869	0,628	11,3	17,6	22,0	79,7	4,7
Luxembourg	0,943	0,721	5,0	12,5	14,3	88,2	-0,6
Hungary	0,879	0,590	8,3	12,4	12,4	84,6	3,4
Malta	0,895	0,531	5,3	13,7	15,5	86,0	3,2
Netherlands	0,954	0,882	4,0	10,5	10,4	75,0	-2,6
Austria	0,930	0,744	5,5	11,2	13,5	72,8	1,5
Poland	0,877	0,631	8,8	17,0	16,7	80,8	4,2
Portugal	0,907	0,753	6,2	17,9	19,1	88,7	-1,8
Romania	0,836	0,512	7,5	22,4	24,3	91,9	2,6
Slovenia	0,927	0,641	7,1	11,0	13,6	91,1	1,5
Slovakia	0,877	0,663	8,1	10,1	11,5	73,6	0,5
Finland	0,954	0,902	6,8	12,7	14,5	78,1	0,8
Sweden	0,956	0,909	4,1	11,3	13,0	85,3	-0,5
United Kingdom	0,943	0,790	4,2	17,5	20,1	72,4	1,2

Taxonomic methods

1. Taxonomic measure of standard of living (TMSL) – the classic Hellwig's approach;
2. Generalized distance measure (GDM) – Walesiak's proposal;
3. Taxonomic measure of intervals (TMI);
4. Taxonomic measure of quotients (TMQ).

Taxonomic measure of gender equality (TMGE)

1. Variable normalization (by standardization)

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{S_j}$$

where:

$\bar{x}_j$ - arithmetic mean for each j variable (j = 1, 2, ..., m);

S_j - standard deviation for each j variable (j = 1, 2, ..., m);

2. Euclidean distance measure is given as:

$$d_{i0} = \sqrt{\sum_{j=1}^{k} \left(z_{ij} - z_{0j}\right)^2 w_j}$$

where:

z_{0j} - normalized values of the artificial object (economy) that have the best possible values for each variable;

Taxonomic measure of gender equality (TMGE)

3. Taxonomic measure of gender equality (TMGE) is determined as follows:

$$TMGE_i = 1 - \frac{d_{i0}}{d_0}$$

where:

d_{i0} - the Euclidean metrics for each given economy;

$$d_0 = \bar{d}_{i0} + 3 \cdot S(d_{i0})$$

$\bar{d}_{i0}$ - average Euclidean metrics measured by countries;

$S(d_{i0})$ - standard deviation of Euclidean metrics.

Results

Rank	EU Coutries	TMGE
1	Sweden	0,873
2	United Kingdom	0,813
3	Denmark	0,805
4	Finland	0,794
5	Netherlands	0,791
6	Luxembourg	0,755
7	Austria	0,709
8	Cyprus	0,704
9	Ireland	0,704
10	Slovenia	0,684
11	France	0,669
12	Belgium	0,657
13	Germany	0,656
14	Spain	0,604
15	Czech Republic	0,601
16	Malta	0,599
17	Portugal	0,530
18	Italy	0,506
19	Greece	0,452
20	Poland	0,439
21	Estonia	0,394
22	Slovakia	0,380
23	Hungary	0,352
24	Bulgaria	0,345
25	Lithuania	0,288
26	Romania	0,261
27	Latvia	0,147

Next directions of activities

- To prove empirically that the 15th stylized fact is true.
- To determine Taxonomic Measure of Gender Equality (using current data and revising the diagnostic variables set).
- To use TMGE as an exogenous variable in growth models.

THANK YOU FOR YOUR ATTENTION!